By applying an argument similar to the proof of Theorem 1 of Section 4.3.2, it follows from the above that (see exercise):

$$\sqrt{n}\left(\widehat{\boldsymbol{\beta}}-\boldsymbol{\beta}\right)=\left(-\frac{\partial}{\partial\boldsymbol{\beta}}\mathbf{w}_n\right)^{-\top}\sqrt{n}\left[\mathbf{w}_n+\left(\frac{\partial}{\partial\boldsymbol{\gamma}}\mathbf{w}_n\right)^{\top}(\widehat{\boldsymbol{\gamma}}-\boldsymbol{\gamma})\right]+ \qquad (4.89)$$
$$+\mathbf{o}_p(1)$$
$$=B^{-1}\frac{\sqrt{n}}{n}\sum_{i=1}^{n}\left(\mathbf{w}_{ni}-CH^{-1}\mathbf{u}_{ni}\right)+\mathbf{o}_p(1)$$

where

$$B=E\left(G_i\Delta_iD_i^{\top}\right),\quad C=E\left[\frac{\partial}{\partial\boldsymbol{\gamma}}\left(G_i(\mathbf{x}_i)\Delta_iS_i\right)\right]^{\top},\quad H=E\left(\frac{\partial}{\partial\boldsymbol{\gamma}}\mathbf{u}_{ni}\right)^{\top}$$

It immediately follows from (4.89) that $\widehat{\boldsymbol{\beta}}$ is asymptotically normal with the asymptotic variance given by (see exercise):

$$\Sigma_\beta=B^{-1}\left[E\left(G_i\Delta_iS_iS_i^{\top}\Delta_iG_i^{\top}\right)+\Phi\right]B^{-\top} \qquad (4.90)$$
$$\Phi=CH^{-1}Var(\mathbf{u}_{ni})H^{-\top}C^{\top}-E\left(\mathbf{w}_{ni}\mathbf{u}_{ni}^{\top}H^{-\top}C^{\top}\right)-\left[E\left(\mathbf{w}_{ni}\mathbf{u}_{ni}^{\top}H^{-\top}C^{\top}\right)\right]^{\top}$$

A consistent estimate of $\Sigma_\beta$ is readily obtained by substituting consistent estimates of the respective quantities in (4.90). In comparison to (4.83), the asymptotic variance has an additional correction factor.

So far, we have assumed that we can model $\pi_{it}$ correctly when it is unknown. If the model for $\pi_{it}$ is wrong, that is, $\pi_{it}\neq E(r_{it}=1\mid H_{it})$, the condition $E(\Delta_iS_i)=\mathbf{0}$ may not be true and the WGEE may yield biased estimates of $\boldsymbol{\beta}$. In some cases, however, we can still consistently estimate $\boldsymbol{\beta}$ if we have a correct model for the missing $y_{it}$. We illustrate this *double robustness* idea again with a relatively simple pre-post study design involving two assessment times.

As noted in Section 4.2.1, ANOVA and ANCOVA may be applied to compare treatment difference at posttreatment for controlled randomized longitudinal trials. Under MAR, these models generally yield biased estimates when applied directly to the observed data at posttreatment. By generalizing Example 5 to multisample repeated measures ANOVA, we can immediately address MAR using WGEE.

**Example 6 (*Multisample repeated measures ANOVA under MAR*).** Within the pre-post study design in Example 5, consider comparing $g$ treatment conditions with $n_k$ subjects in the $k$th treatment condition